Sample Question Paper - 12
Mathematics-Basic (241)
Class- X, Session: 2021-22
TERM II

Time Allowed: 120 minutes **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
2. All questions are compulsory.
3. Section A comprises of 6 questions of 2 marks each. Internal choice has been provided in two questions.
4. Section B comprises of 4 questions of 3 marks each. Internal choice has been provided in one question.
5. Section C comprises of 4 questions of 4 marks each. An internal choice has been provided in one question. It contains two case study based questions.

SECTION A

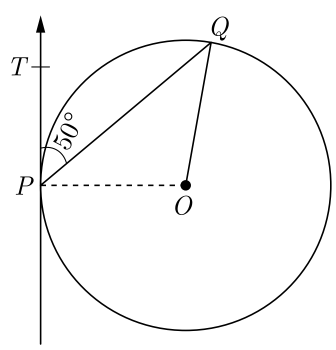
1. If 4 times the 4^{th} term of an AP is equal to 18 times the 18^{th} term, then find the 22^{nd} term.

OR

In an A.P., if the common difference $d = -3$ and the eleventh term $a_{11} = 15$, then find the first term.

2. From a point on the ground, 20 m away from the foot of a vertical tower, the angle of elevation of the top of the tower is 60°. Find the height of the tower.

3. In figure, O is the centre of circle. PQ is a chord and PT is tangent at P which makes an angle of 50° with PQ . Find the angle $\angle POQ$.



4. A solid piece of iron in the form of a cuboid of dimensions $49\,\text{cm} \times 33\,\text{cm} \times 24\,\text{cm}$, is moulded to form a solid sphere. What is the radius of the sphere ?

5. A solid metallic cuboid 24 cm $\times$ 11 cm $\times$ 7 cm is melted and recast into solid cones of base radius 3.5 cm and height 6 cm. Find the number of cones so formed.

6. Find the mode of the following grouped frequency distribution.

Class	**Frequency**
3-6	2

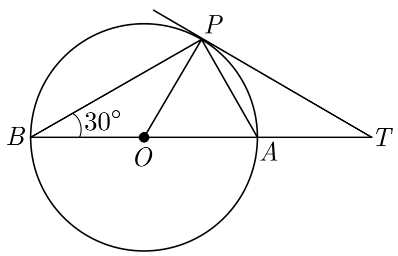
Class	Frequency
6-9	5
9-12	10
12-15	23
15-18	21
18-21	12
21-24	03

OR

If the mean of the first n natural number is 15, then find n.

Section B

7. In the given figure, BOA is a diameter of a circle and the tangent at a point P meets BA when produced at T. If $\angle PBO = 30^0$, what is the measure of $\angle PTA$?



8. A cone of height 36 cm and radius of base 9 cm is made up of moulding clay. A child reshapes it in the form of a sphere. Find the diameter of the sphere.

9. Find the mode of the following frequency distribution:

Class	15-20	20-25	25-30	30-35	35-40	40-45
Frequency	3	8	9	10	3	2

10. The weekly expenditure of 500 families is tabulated below :

Weekly Expenditure(Rs.)	Number of families
0-1000	150
1000-2000	200
2000-3000	75
3000-4000	60
4000-5000	15

Find the median expenditure.

OR

Prove that $\sum(x_i - \overline{x}) = 0$

Section C

11. If the sum of first four terms of an AP is 40 and that of first 14 terms is 280. Find the sum of its first n terms.

12. Draw two concentric circles of radii 2 cm and 5 cm. Take a point P on the outer circle and construct a pair of tangents PA and PB to the smaller circle. Measure PA.

OR

Draw a circle of radius 3 cm. From a point P, 7 cm away from centre draw two tangents to the circle. Measure the length of each tangent.

13. Maximum Profit : A kitchen utensils manufacturer can produce up to 200 utensils per day. The profit made from the sale of these utensils can be modelled by the function $P(x) = -0.5x + 175x - 330$, where $P(x)$ is the profit in Rupees, and x is the number of utensils made and sold. Based on this model,

(i) Find the x-intercepts and explain what they mean in this context.

(ii) How many utensils should be sold to maximize profit?

14. Irrigation Canals : Irrigation canals are the main waterways that bring irrigation water from a water source to the areas to be irrigated. The water is taken either from the river, tank or reservoirs. The canals can be constructed either by means of concrete, stone, brick or any sort of flexible membrane which solves the durability issues like seepage and erosion.

One such canal shown above is of width 5.4 m wide and depth 1.8 m deep through which water is flowing with a speed of 25 km/hour.

(i) How much water is flowing through the canal in 1 hour.

(ii) At some distance from canal, a framer is having a large cylindrical tank the radius of whose base is 2 m. Suppose the farmer connects this tank to canal by a circular pipe of internal diameter of 4 cm for irrigation his field. If water is flowing at 7 m/s through a circular pipe, find the increase in water level in 30 minutes.

Solution

MATHEMATICS BASIC 241

Class 10 - Mathematics

Time Allowed: 120 minutes **Maximum Marks: 40**

General Instructions:

1. The question paper consists of 14 questions divided into 3 sections A, B, C.
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SECTION A

1. If 4 times the 4^{th} term of an AP is equal to 18 times the 18^{th} term, then find the 22^{nd} term.

Sol :

Let a be the first term and d be the common difference of the AP.

Now $a_n = a + (n-1)d$

As per the information given in question

$$4 \times a_4 = 18 \times a_{18}$$
$$4(a+3d) = 18(a+17d)$$
$$2a+6d = 9a+153d$$
$$7a = -147d$$
$$a = -21d$$
$$a+21d = 0$$
$$a+(22-1)d = 0 \Rightarrow a_{22} = 0$$

Hence, the 22^{nd} term of the AP is 0.

or

In an A.P., if the common difference $d = -3$ and the eleventh term $a_{11} = 15$, then find the first term.

Sol :

We have, $a_{11} = 15$ and $d = -3$

Now, $a_n = a + (n-1)d$

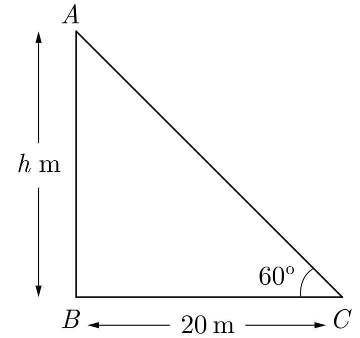
$$a_{11} = a + (11-1$$
$$a_{11} = a + 10d$$
$$15 = a + 10(-3)$$
$$15 = a - 30$$
$$a = 45$$

So, first term is 45.

2. From a point on the ground, 20 m away from the foot of a vertical tower, the angle of elevation of the top of the tower is 60°. Find the height of the tower.

Sol :

Let h be the height of tower. Now as per question we have shown the figure below.

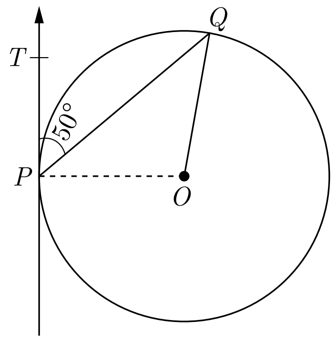


In ΔABC, $\tan 60^\circ = \frac{AB}{BC}$

$$\sqrt{3} = \frac{h}{20}$$
$$h = 20\sqrt{3} \text{ m}$$

3. In figure, O is the centre of circle. PQ is a chord and PT is tangent at P which makes an angle of 50°

with PQ. Find the angle $\angle POQ$.



Sol :

Due to angle between radius and tangent,

$$\angle OPT = 90^\circ$$

$$\angle OPQ = 90^\circ - 50^\circ = 40^\circ$$

Also, $OP = OQ$ [Radii of a circle]

Since equal opposite sides have equal opposite angles,

$$\angle OPQ = \angle OQP = 40^\circ$$

$$\angle POQ = 180^\circ - \angle OPQ - \angle OQP$$

$$= 180^\circ - 40^\circ - 40^\circ = 100^\circ$$

4. A solid piece of iron in the form of a cuboid of dimensions $49\text{ cm} \times 33\text{ cm} \times 24\text{ cm}$, is moulded to form a solid sphere. What is the radius of the sphere ?

Sol :

Volume of the sphere = Volume of the cuboid

$$\frac{4}{3}\pi r^3 = 49 \times 33 \times 24 = 38808\text{ cm}^3$$

$$4 \times \frac{22}{7} r^3 = 38808 \times 3$$

$$r^3 = \frac{38808 \times 3 \times 7}{4 \times 22}$$

$$= 441 \times 21$$

$$r^3 = 21 \times 21 \times 21$$

$$r = 21\text{ cm}$$

5. A solid metallic cuboid 24 cm $\times$ 11 cm $\times$ 7 cm is melted and recast into solid cones of base radius 3.5 cm and height 6 cm. Find the number of cones so formed.

Sol :

Let n be the number of cones formed.

Now, according to question,

Volume of n cones = Volume of cuboid

$$n \times \frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times 6 = 24 \times 11 \times 7$$

$$n = \frac{24 \times 11 \times 7 \times 3 \times 7}{22 \times 3.5 \times 3.5 \times 6} = 24$$

Thus $n = 24$.

6. Find the mode of the following grouped frequency distribution.

Class	Frequency
3-6	2
6-9	5
9-12	10
12-15	23
15-18	21
18-21	12
21-24	03

Sol :

We observe that the class 12-15 has maximum frequency 23. Therefore, this is the modal class.

We have, $l = 12$, $h = 3$, $f_1 = 23$, $f_0 = 10$ and $f_2 = 21$

$$M_o = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 12 + \frac{23 - 10}{46 - 10 - 21} \times 3$$

$$= 12 + \frac{13}{15} \times 3$$

$$= 12 + \frac{13}{5} = 14.6$$

or

If the mean of the first n natural number is 15, then find n.

Sol :

Given : 1, 2, 3, 4, ... to n terms.

The sum of first n natural numbers

$$S_n = \frac{n(n+1)}{2}$$

Mean, $$M = \frac{n(n+1)}{2 \times n}$$

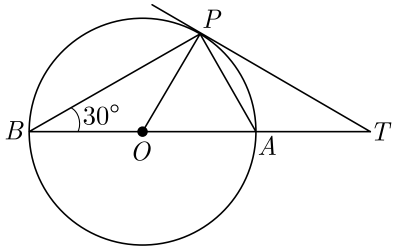
$$15 = \frac{n(n+1)}{2 \times n}$$

$$15 = \frac{n+1}{2}$$

$$n + 1 = 30 \Rightarrow n = 29$$

Section B

7. In the given figure, BOA is a diameter of a circle and the tangent at a point P meets BA when produced at T. If $\angle PBO = 30^0$, what is the measure of $\angle PTA$?



Sol :

Angle inscribed in a semicircle is always right angle.

$$\angle BPA = 90^0$$

Here OB and OP are radius of circle and equal in length, thus angle $\angle OBP$ and $\angle OPB$ are also equal.

Thus $\angle BPO = \angle PBO = 30^0$

Now $\angle POA = \angle OBP + \angle OPB$

$$= 30^\circ + 30^\circ = 60^\circ$$

Thus $\angle POT = \angle POA = 60^\circ$

Since OP is radius and PT is tangent at P, $OP \perp PT$

$$\angle OPT = 90^0$$

Now in right angle ΔOPT,

$$\angle PTO = 180^\circ - (\angle OPT + \angle POT)$$

Substituting $\angle OPT = 90^0$ and $\angle POT = 60^\circ$ we have

$$\angle PTO = 180^\circ - (90^\circ + 60^\circ)$$

$$= 180^\circ - 150^\circ$$

$$= 30^o$$

Thus $\angle PTA = \angle PTO = 30^o$

8. A cone of height 36 cm and radius of base 9 cm is made up of moulding clay. A child reshapes it in the form of a sphere. Find the diameter of the sphere.

Sol :

Let R be the radius of sphere. For the cone we have

Height, $h = 36\text{ cm}$

Radius, $r = 9\text{ cm}$

Now, according to the question

Volume of sphere = Volume of cone

$$\frac{4}{3}\pi R^3 = \frac{1}{3}\pi r^2 h$$

$$4R^3 = r^2 h$$

$$4R^3 = (9)^2 \times 36$$

$$R^3 = \frac{9 \times 9 \times 36}{4}$$

$$= 9 \times 9 \times 9 = 9^3$$

$$R = 9\text{ cm}$$

9. Find the mode of the following frequency distribution:

Class	15-20	20-25	25-30	30-35	35-40	40-45
Frequency	3	8	9	10	3	2

Sol :

Class 30-35 has the maximum frequency 10, therefore this is model class.

Now $l = 30,\ f_0 = 9,\ f_1 = 10,\ f_2 = 3$ and $h = 5$

Mode, $M_o = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)h$

$$= 30 + \left(\frac{10-9}{2 \times 10 - 9 - 3}\right. \times 5$$

$$= 30 + \frac{5}{8}$$

$$= 30 + 0.625 = 30.625$$

10. The weekly expenditure of 500 families is tabulated below :

Weekly Expenditure(Rs.)	Number of families
0-1000	150
1000-2000	200
2000-3000	75
3000-4000	60
4000-5000	15

Find the median expenditure.

Sol :

We prepare following cumulative frequency table to find median class.

Expenditure	f (families)	c.f.
0-1000	150	150
1000-2000	200	350
2000-3000	75	425

3000-4000	60	485
4000-5000	15	500
	$\sum f = 500$	

We have $\quad N = 500 \; ; \; \frac{N}{2} = 250$

Cumulative frequency just greater than $\frac{N}{2}$ is 350 and the corresponding class is 1000-2000. Thus median class is 1000-2000.

Now, $l = 1000, \; f = 200, \; F = 150$ and $h = 1000$

$$\text{Median,} \quad M_d = l + \left(\frac{\frac{N}{2} - F}{f}\right)h$$

$$= 1000 + \frac{250 - 150}{200} \times 1000$$

$$= 1000 + 500 = 1,500$$

Thus median expenditure is Rs. 1500 per week.

or

Prove that $\sum (x_i - \overline{x}) = 0$

Sol :

We have $\quad \overline{x} = \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)$

$$n\overline{x} = \sum_{i=1}^{n} x_i$$

Now, $\quad \sum_{i=1}^{n}(x_i - \overline{x}) = (x_1 - \overline{x}) + (x_2 - \overline{x}) + \dots + (x_n - \overline{x})$

$$\sum_{i=1}^{n}(x_i - \overline{x}) = (x_1 + x_2 + \dots + x_n) - n\overline{x}$$

$$\sum_{i=1}^{n}(x_i - \overline{x}) = \sum_{i=1}^{n} x_i - n\overline{x} = \sum_{i=1}^{n}(x_i - \overline{x})$$

Hence, $\quad \sum_{i=1}^{n}(x_i - \overline{x}) = 0$

Section C

11. If the sum of first four terms of an AP is 40 and that of first 14 terms is 280. Find the sum of its first n terms.

Sol :

Let a be the first term and d be the common difference.

Sum of n terms of an AP,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Now $S_4 = 40$ and $S_{14} = 280$

$$\frac{4}{2}[2a + (4-1)d] = 40$$

$$2[2a + 3d] = 40$$

$$2a + 3d = 20$$

(1)

and $\quad \frac{14}{2}[2a + (14-1)d] = 280$

$$7[2a + 13d] = 280$$

$$2a + 13d = 40 \quad (2)$$

Solving equations (1) and (2), we get

$$a = 7 \text{ and } d = 2$$

Now $\quad S_n = \frac{n}{2}[2 \times 7 + (n-1)2]$

$$= \frac{n}{2}[14 + 2n - 2]$$

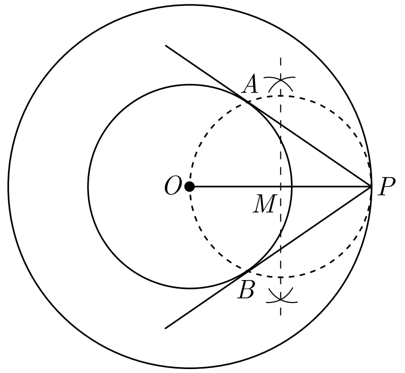
$$= \frac{n}{2}(12 + 2n) = 6n + n^2$$

Hence, sum of n terms is $6n + n^2$.

12. Draw two concentric circles of radii 2 cm and 5 cm. Take a point P on the outer circle and construct a pair of tangents PA and PB to the smaller circle. Measure PA.

Sol :

1. Draw a circle with centre O and radius 2 cm.
2. Draw another circle with centre O and radius 5 cm.
3. Take a point P on outer circle and join OP.
4. Draw perpendicular bisector of OP which intersect OP at M.
5. Draw a circle with centre M which intersects inner circle at points A and B.
6. Join AP and BP. Thus AP and BP are required tangents.



$$PA = \sqrt{5^2 - 2^2}$$

$= \sqrt{21} = 4.6$ cm

or

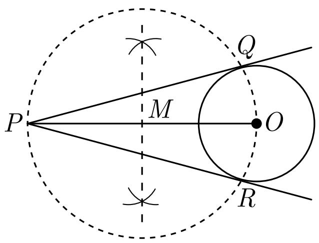
Draw a circle of radius 3 cm. From a point P, 7 cm away from centre draw two tangents to the circle. Measure the length of each tangent.

Sol :

Steps of Construction :

1. Draw a line segment PO of length 7 cm.
2. Draw a circle with centre O and radius 3 cm.
3. Draw a perpendicular bisector of PO. Let M be the mid-point of PO.
4. Taking M as centre and OM as radius draw a circle. Let this circle intersects the given circle at the point Q and R.
5. Join PQ and PR. On measuring we get

$$PQ = PR = 6.3 \text{ cm.}$$



13. Maximum Profit : A kitchen utensils manufacturer can produce up to 200 utensils per day. The profit made from the sale of these utensils can be modelled by the function $P(x) = -0.5x + 175x - 330$, where $P(x)$ is the profit in Rupees, and x is the number of utensils made and sold. Based on this model,

(i) Find the x-intercepts and explain what they mean in this context.

(ii) How many utensils should be sold to maximize profit?

Sol :

We have $\quad P(x) = -0.5x^2 + 175x - 3300$

(i) If there is no profit, i.e. $P(x) = 0$

$$-0.5x^2 + 175x - 3300 = 0$$

$$-0.5(x^2 - 350x + 6600) = 0$$

$$-0.5(x - 20)(x - 330) = 0$$

$$x = 20, 330; \ x = 330$$

Thus (20, 0) and (330, 0). If less than 20 or more than 330 utensils are made sold, there will be no profit. Thus x intercept is break even point because $P(x) = 0$ it is plotted on x.

(ii) For maximum profit,

$$P(x) = -0.5x^2 + 175x - 3300$$

$$= -0.5(x^2 - 350x) - 3300$$

$$= -0.5(x^2 - 350x + 175^2 - 175^2) - 3300$$

$$= -0.5[(x - 175)^2 - 30625] - 3300$$

$$= -0.5(x - 175)^2 + 15312.5 - 3300$$

or $\quad P(x) = -0.5(x - 175)^2 + 12012.5$

From above equation it is clear that $P(x)$ is maximum at $x = 175$ and this maximum value is 12012.5.

14. Irrigation Canals : Irrigation canals are the main waterways that bring irrigation water from a water source to the areas to be irrigated. The water is taken either from the river, tank or reservoirs. The canals can be constructed either by means of concrete, stone, brick or any sort of flexible membrane which solves the durability issues like seepage and erosion.

One such canal shown above is of width 5.4 m wide and depth 1.8 m deep through which water is flowing with a speed of 25 km/hour.

(i) How much water is flowing through the canal in 1 hour.

(ii) At some distance from canal, a framer is having a large cylindrical tank the radius of whose base is 2 m. Suppose the farmer connects this tank to canal by a circular pipe of internal diameter of 4 cm for irrigation his field. If water is flowing at 7 m/s through a circular pipe, find the increase in water level in 30 minutes.

Sol :

(i) Water flow in 1 hour,

$$= \text{Area of cross-section} \times \text{Speed of water}$$

$$= 5.4 \times 1.8 \times 25000 \text{ m}^3$$

$$= 54 \times 18 \times 250 \text{ m}^3$$

$$= 243000 \text{ m}^3$$

(ii) Length of water that flows from circular pipe in 1 sec is 7 m or 700 cm.

Radius of pipe is $\frac{4}{2} = 2$ cm.

Thus volume of water in 1 second,

$$= \pi \times (2)^2 \times 700 \text{ cm}^3$$

Volume of water in 30 minutes,

$$= \pi \times (2)^2 \times 700 \times 60 \times 30 \text{ cm}^3$$

Let h be height of water in tank. Since base of tank is 2 m, radius of tank is 1 m i.e. 100 cm.

Volume of water in the tank,

$$\pi 100^2 \times h = \pi \times 4 \times 700 \times 60 \times 30$$

$$h = \frac{700 \times 60 \times 30 \times 4}{100 \times 100} = 504 \text{ cm}$$

Hence, water level increased is 504 cm or 5.04 m.